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On Bayesian Estimation of Loss and Risk Functions for Exponential Distribution

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Abstract

Loss functions and Risk functions are two very important aspects in Bayesian estimation. In this paper the Bayesian estimation for the loss and risk functions of the unknown parameter θ of exponential distribution has been studied under Rukhin's loss function for Type II censoring. The inverse Gamma distribution has been assumed as the prior distribution for the unknown parameter θ. On the part of loss functions, the Squared Error Loss Function (SELF) and three different forms of Weighted Squared Error Loss Function (WSELF) namely, the Degroot's Loss Function, Minimum Expected Loss (MELO) Function and Exponentially Weighted Minimum Expected Loss (EWMELO) Function have been considered.

Keywords: ***Bayes Estimator; Loss function; Risk function; Exponential distribution; Type II censoring***

1. Introduction

A continuous random variable X is said to have Exponential distribution, if its probability density function is given by,

$$f(x,\theta) = \begin{cases} \frac{e^{-\frac{x}{\theta}}}{\theta}, \text{if } x > 0, \theta > 0 \\ 0, \text{Otherwise.} \end{cases} \quad (1.1)$$

This distribution is one of the most important distributions having it application specially in life testing. In this case,

$$E(X) = \theta \quad (1.2)$$

For specified mission time$'$ t$'$, the reliability of the distribution, denoted by $R(t, \theta)$, is given by

$$R(t,\theta) = P(X \geq t) = e^{-\frac{t}{\theta}} \qquad (1.3)$$

The failure rate for this distribution, denoted by
$h(t) = \frac{f(t,\theta)}{R(t,\theta)} = \frac{1}{\theta}$, a constant.

Due to constant failure rate this distribution is most suitable for items which are free from the effect of so called 'Ageing', specially electrical and electronic items.

2. Basic Concepts

Rukhin [1] proposed a loss function which is given by

$$L(\theta,\delta,\gamma) = w(\theta,\delta)\gamma^{-\frac{1}{2}} + \gamma^{\frac{1}{2}} \qquad (2.1)$$

Where γ is an estimator of the loss function $w(\theta,\delta)$ which is non-negative.
$\frac{\partial L(\theta,\delta,\gamma)}{\partial\gamma} = 0$ gives $\gamma = w(\theta,\delta)$

Since $\frac{\partial^2 L(\theta,\delta,\gamma)}{\partial\gamma^2} > 0$ at $\gamma = w(\theta,\delta)$ .So $L(\theta,\delta,\gamma)$ is minimum in γ when $\gamma = w(\theta,\delta)$.

Let $\underline{X} = (X_1, X_2, \dots X_n)$ be a random sample of size n, then the Bayes risk is

$$E\{\, L(\theta,\delta,\gamma)/\underline{X}\} = E\{w(\theta,\delta)/\underline{X}\}\gamma^{-\frac{1}{2}} + \gamma^{\frac{1}{2}}$$

$\frac{\partial E\{L(\theta,\delta,\gamma)/\underline{X}\}}{\partial\gamma} = 0$ gives $\gamma = E\{w(\theta,\delta)/\underline{X}\}$.

So, if $\delta_B(\underline{X})$ is the Bayes estimator of θ or a function of θ under the loss function $w(\theta,\delta)$ $\gamma_B(X)$ be the Bayes estimator of θ corresponding to $L(\theta,\delta,\gamma)$.Thus,

$$\gamma_B(\underline{X}) = E\{w(\theta,\delta_B)/\underline{X}\} \qquad (2.2)$$

A generalized form denoted by $L(\phi(\theta),\delta,\gamma)$ for estimating a function $\phi(\theta)$,is given by,

$$L(\phi(\theta),\delta,\gamma) = w(\phi(\theta),\delta)\gamma^{-\frac{1}{2}} + \gamma^{\frac{1}{2}} \quad (2.3)$$

Where, γ is an estimator of the loss function $w(\phi(\theta),\delta)$.

The Bayes estimator of $\phi(\theta)$ corresponding to $L(\phi(\theta),\delta,\gamma)$,denoted by $\gamma_{\phi B}(\underline{X})$,is given by,

$$\gamma_{\phi B}(\underline{X}) = E\{w(\phi(\theta), \phi_B)/\underline{X}\} \quad (2.4)$$

Where, $\phi_B(\underline{X})$ is the Bayes estimator of $\phi(\theta)$ under the loss function $w(\phi(\theta), \delta)$

Let $X_1, X_2, X_3, \ldots X_n$ be a random sample of size n and $X_{(1)} < X_{(2)} < X_{(3)} < \ldots < X_{(n-1)} < X_{(n)}$ be the order statistic corresponding to this random sample. In case of type- II censoring, n items are placed on test and the test is terminated after first 'r' (r pre-specified) failures. Thus, only r ordered observed values of $X_{(1)} < X_{(2)} < X_{(3)} < \ldots < X_{(r-1)} < X_{(r)}$ are recorded. For observed values $x_{(1)} < x_{(2)} < x_{(3)} < \ldots < x_{(r-1)} < x_{(r)}$,the likelihood function, denoted by $L(\theta)$,is given by,

$$L(\theta) = k\theta^{-r}e^{-\frac{t_r}{\theta}} \quad (2.5)$$

Where, k is function of n, r and $x_{(i)}$ i=1,2…r and does not contain θ.

$$t_r = \sum_{i=1}^{r} x_{(i)} + (n-r)x_{(r)} \quad (2.6)$$

t_r is an observed value of the statistic T_r given by,

$$T_r = \sum_{i=1}^{r} X_{(i)} + (n-r)X_{(r)} \quad (2.7)$$

Epstein and Sobel [2] have proved that $\frac{T_r}{r}$ is M.L. E as well as UMVUE for θ.This estimator also attains the Cramér-Rao Lower bound. Bhattacharya [3] derived the estimate of θ and $R(t, \theta)$ in Bayesian framework under the assumption of Squared Error Loss Function (SELF) and three different prior distributions for θ. Guobing Fan [4] has derived the Bayes estimator of the loss and risk function of Maxwell's distribution using inverse Gamma distribution as the prior distribution for θ and squared error loss function under the criterion of loss function proposed by Rukhin [1].

In this paper Bayesian estimation of θ and $R(t, \theta)$has been considered under the assumption of inverse Gamma distribution as the prior distribution for the unknown parameter θ and Type II censoring.

On the part of loss function, we have considered the four forms of $w(\theta, \delta)$ and compare their performance. The forms to be considered are as follows:

1. Squared Error Loss Function (SELF): In this case,

$$w(\theta, \delta) = (\theta - \delta)^2 \quad (2.8)$$

This loss function is symmetric but unbounded. It suffers from the drawback of giving the same weight to overestimation as well as to underestimation.

2. $w(\theta, \delta) = \delta^{-2}(\theta - \delta)^2$ (2.9)

This loss function, introduced by DeGroot [5], is asymmetric. It gives more weight to underestimation than to overestimation.

3. Minimum Expected Loss (MELO) Function: In this case,

$$w(\theta,\delta) = \theta^{-2}(\theta-\delta)^2 \quad (2.10)$$

This loss function is asymmetric and bounded. This loss function was used by Tummala and Sathe [6] for estimating reliability of certain life time distributions and by Zellner [7] for estimating functions of parameters in econometric models.

4. Exponentially Weighted Minimum Expected Loss (EWMELO) Function

$$w(\theta,\delta) = \theta^{-2}e^{-a\theta^{-1}}(\theta-\delta)^2 \quad (2.11)$$

This loss function is also asymmetric and bounded. This type of loss function was used by the author [8] for the first time in his work for D.Phil. SELF, MELO and EWMELO were used by Singh, the author, [9] in the study of reliability of a multicomponent system and in Bayesian Estimation of the mean and distribution function of Maxwell's distribution. Recently [10], the author again used these loss functions in Bayesian estimation of function of the unknown parameter θ for the Modified Power Series Distribution (MPSD) and for estimating Loss and Risk Functions of a continuous distribution. Details of other works done by the author are given in [11] to [16] in the references.

3. Bayesian Estimation of Loss and Risk Function

In this section, the estimation of the loss function has been performed for the probability density function specified by (1) under various forms of $w(\theta,\delta)$, given by (2.8), (2.9), (2,10) and (2.11) respectively. Results are given in the following:

Theorem1. Let $\underline{X} = (X_1, X_2, \ldots X_n)$ be a random sample of size n and $X_{(1)} < X_{(2)} < X_{(3)} < \ldots < X_{(n-1)} < X_{(n)}$ be the order statistic corresponding to this random sample from the exponential distribution, specified by the probability density function given by (1). Under the assumption of inverse Gamma distribution as the prior distribution for θ,Bayes estimators of θ corresponding to various loss functions given as above and Bayes estimators of $w(\theta,\delta)$ based on Rukhin's loss function given by (2.1) are as follows:

(a) When $w(\theta,\delta) = (\theta-\delta)^2$,

$$\hat{\theta}_B = \frac{\beta+T_r}{r+\alpha-1} \quad (3.1)$$

$$\gamma_B(\underline{X}) = \frac{(\beta+T_r)^2}{(r+\alpha-2)(r+\alpha-1)^2} \quad (3.2)$$

$$E_\theta\{\gamma_B(\underline{X})\} = \frac{r\theta^2+(r\theta+\beta)^2}{(r+\alpha-2)(r+\alpha-1)^2} \quad (3.3)$$

The risk function of $\hat{\theta}_B$ is,

$$R(\theta,\hat{\theta}_B) = \frac{r\theta^2+\{(1-\alpha)\theta+\beta\}^2}{(r+\alpha-1)^2} \quad (3.4)$$

(b) When $w(\theta,\delta) = \delta^{-2}(\theta-\delta)^2$

$$\hat{\theta}_D = \frac{\beta+T_r}{r+\alpha-2} \quad (3.5)$$

$$\gamma_D(\underline{X}) = \frac{1}{(r+\alpha-1)} \quad (3.6)$$

$$E_\theta\{\gamma_D(\underline{X})\} = \frac{1}{(r+\alpha-1)} \quad (3.7)$$

$$R(\theta, \hat{\theta}_D) = 1 + \frac{\theta^2\{r\theta^2+(r\theta+\beta)^2\}}{(r+\alpha-2)^2} - 2\theta(r+\alpha-2)E_\theta\left(\frac{1}{\beta+T_r}\right) \quad (3.8)$$

(c) When $w(\theta,\delta) = \theta^{-2}(\theta-\delta)^2$

$$\hat{\theta}_M = \frac{\beta+T_r}{r+\alpha+1} \quad (3.9)$$

$$\gamma_M(\underline{X}) = \frac{1}{(r+\alpha+1)} \quad (3.10)$$

$$E_\theta\{\gamma_M(\underline{X})\} = \frac{1}{(r+\alpha+1)} \quad (3.11)$$

$$R(\theta, \hat{\theta}_M) = \frac{r+\{\beta\theta^{-1}-(\alpha+1)\}^2}{(r+\alpha+1)^2} \quad (3.12)$$

(d)When $w(\theta,\delta) = \theta^{-2}e^{-a\theta^{-1}}(\theta-\delta)^2, a > 0$

$$\hat{\theta}_E = \frac{\beta+T_r+a}{r+\alpha+1} \quad (3.13)$$

$$\gamma_E(\underline{X}) = \frac{1}{(r+\alpha+1)}\left(\frac{\beta+T_r}{\beta+T_r+a}\right)^{r+\alpha} \quad (3.14)$$

$$E_\theta\{\gamma_E(\underline{X})\} = \frac{1}{(r+\alpha+1)}E\left\{\left(\frac{\beta+T_r}{\beta+T_r+a}\right)^{r+\alpha}\right\} \quad (3.15)$$

$$R(\theta, \hat{\theta}_E) = \frac{e^{-a\theta^{-1}}\{r+\left(\beta\theta^{-1}+a\theta^{-1}-\alpha-1\right)^2\}}{(r+\alpha+1)^2} \quad (3.16)$$

Proof: Let the prior probability density function of θ, which is inverse gamma distribution, be given by,

$$\pi(\theta) = \begin{cases} \frac{\beta^\alpha\theta^{-(\alpha+1)}e^{-\frac{\beta}{\theta}}}{\Gamma(\alpha)}, \text{if } \theta > 0, \alpha, \beta > 0 \\ 0, \text{Otherwise.} \end{cases} \quad (3.17)$$

Where $\alpha, \beta > 0$ are known.

Let t_r be an observed value of the statistic T_r .Then, the posterior probability density function of θ is given by,

$$\pi(\theta / t_r) = \begin{cases} \frac{(\beta+t_r)^{\alpha+r}\theta^{-(\alpha+r+1)}e^{-\frac{(\beta+t_r)}{\theta}}}{\Gamma(\alpha+r)}, \text{if } \theta > 0, \alpha, \beta > 0 \\ 0, \text{Otherwise.} \end{cases} \quad (3.18)$$

(a) When $w(\theta,\delta) = (\theta-\delta)^2$,the Bayes estimator of θ,denoted by $\hat{\theta}_B$ is the mean of the posterior distribution and is given by,

$$\hat{\theta}_B = E(\theta / T_r) = \frac{\beta+T_r}{r+\alpha-1}$$

$$\gamma_B(\underline{X}) = E\{w(\theta,\hat{\theta}_B)/\underline{X}\} = E\{(\theta-\hat{\theta}_B)^2/T_r\} =$$

$$Var(\theta / T_r) = \frac{(\beta+T_r)^2}{(r+\alpha-2)(r+\alpha-1)^2}$$

$$E_\theta\{\gamma_B(\underline{X})\} = \frac{E_\theta\{(\beta+T_r)^2\}}{(r+\alpha-2)(r+\alpha-1)^2} = \frac{r\theta^2+(r\theta+\beta)^2}{(r+\alpha-2)(r+\alpha-1)^2}$$

$$R(\theta,\hat{\theta}_B) = E_\theta\{w(\theta,\hat{\theta}_B)\} = E_\theta\left\{(\theta - \frac{\beta+T_r}{r+\alpha-1})^2\right\} = \frac{\theta^2\{r+(\alpha-1)^2\}+2\beta(1-\alpha)\theta+\beta^2}{(r+\alpha-1)^2} = \frac{r\theta^2+\{(1-\alpha)\theta+\beta\}^2}{(r+\alpha-1)^2}$$

(b) When $w(\theta,\delta)=\delta^{-2}(\theta-\delta)^2$,the Bayes estimator of θ,denoted by $\hat{\theta}_D$ is given by,

$$\hat{\theta}_D=\frac{E(\theta^2\ /\ T_r)}{E(\theta\ /\ T_r)}=\frac{\beta+T_r}{r+\alpha-2}$$

$$\gamma_D(\underline{X})=E\{w(\theta,\hat{\theta}_D)/\underline{X}\}=E\{\hat{\theta}_D^{-2}(\theta-\hat{\theta}_D)^2/T_r\}=\frac{1}{r+\alpha-1}$$

Since, $\frac{1}{r+\alpha-1}$ is a constant, $E_\theta\{\gamma_D(\underline{X})\}=\frac{1}{(r+\alpha-1)}$

$$R(\theta,\hat{\theta}_D)=E_\theta\{\ w(\theta,\hat{\theta}_D)\}=E_\theta\{\hat{\theta}_D^{-2}(\theta-\hat{\theta}_D)^2\ \}$$

$$R(\theta,\hat{\theta}_D)=1+\frac{\theta^2\{r\theta^2+(r\theta+\beta)^2\}}{(r+\alpha-2)^2}-2\theta(r+\alpha-2)E_\theta(\frac{1}{\beta+T_r})$$

(c) When $w(\theta,\delta)=\theta^{-2}(\theta-\delta)^2$,the Bayes estimator of θ,denoted by $\hat{\theta}_M$ is given by,

$$\hat{\theta}_M=\frac{E(\theta^{-1}\ /\ T_r)}{E(\theta^{-2}\ /\ T_r)}=\frac{\beta+T_r}{r+\alpha+1}$$

$$\gamma_M(\underline{X})=E\{w(\theta,\hat{\theta}_M)/\underline{X}\}=E\{\theta^{-2}(\theta-\hat{\theta}_M)^2/T_r\}=\frac{1}{r+\alpha+1}$$

Since, $\frac{1}{r+\alpha+1}$ is a constant, $E_\theta\{\gamma_M(\underline{X})\}=\frac{1}{(r+\alpha+1)}$

$$R(\theta,\hat{\theta}_M)=E_\theta\{\ w(\theta,\hat{\theta}_M)\}=E_\theta\left\{\theta^{-2}(\theta-\frac{\beta+T_r}{r+\alpha-1})^2\ \right\}$$

$$=\frac{r+(\beta\theta^{-1}-\alpha-1)^2}{(r+\alpha+1)^2}$$

(d)When $w(\theta,\delta)=\theta^{-2}e^{-a\theta^{-1}}(\theta-\delta)^2$, $a>0$, the Bayes estimator of θ,denoted by $\hat{\theta}_E$ is given by,

$$\hat{\theta}_E=\frac{E(\theta^{-1}e^{-a\theta^{-1}}\ /\ T_r)}{E(\theta^{-2}e^{-a\theta^{-1}}\ /\ T_r)}=\frac{\beta+T_r+a}{r+\alpha+1}$$

$$\gamma_E(\underline{X})=E\{w(\theta,\hat{\theta}_E)/\underline{X}\}=E\{\theta^{-2}e^{-a\theta^{-1}}(\theta-\hat{\theta}_E)^2/T_r\}=\frac{1}{r+\alpha+1}E_\theta\{\left(\frac{\beta+T_r}{\beta+T_r+a}\right)^{r+\alpha}\}$$

So, $E_\theta\{\gamma_E(\underline{X})\}=\frac{1}{(r+\alpha+1)}E_\theta\left\{\left(\frac{\beta+T_r}{\beta+T_r+a}\right)^{r+\alpha}\right\}$

Since, $0<\left(\frac{\beta+T}{\beta+T+a}\right)^{r+\alpha}<1$, $0<\ E_\theta\left\{\left(\frac{\beta+T_r}{\beta+T_r+a}\right)^{S+\alpha}\right\}<1$.Therefore,

$$E_\theta\{\gamma_E(\underline{X})\}<\frac{1}{(r+\alpha+1)}=E_\theta\{\gamma_M(\underline{X})\}$$

Since, $\frac{1}{(r+\alpha+1)}<\frac{1}{(r+\alpha-1)}$,we have, $E_\theta\{\gamma_M(\underline{X})\}<E_\theta\{\gamma_D(\underline{X})\}$

Therefore, $E_\theta\{\gamma_E(\underline{X})\}<E_\theta\{\gamma_M(\underline{X})\}<E_\theta\{\gamma_D(\underline{X})\}$

$$R(\theta,\hat{\theta}_E)=E_\theta\{\ w(\theta,\hat{\theta}_E)\}=E_\theta\left\{\theta^{-2}e^{-a\theta^{-1}}(\theta-\frac{\beta+T_r+a}{r+\alpha+1})^2\ \right\}$$

$$=\frac{e^{-a\theta^{-1}}\{r+\left(\beta\theta^{-1}+a\theta^{-1}-\alpha-1\right)^2\}}{(r+\alpha+1)^2}$$

Theorem2. Let $\underline{X}=(X_1,X_2,\ldots X_n)$ be a random sample of size n and$X_{(1)}<\ X_{(2)}<\ X_{(3)}<\ldots<X_{(n-1)}<X_{(n)}$ be the order statistic corresponding to this random sample from the exponential distribution, specified by the probability density function given by (1).Under the assumption of inverse Gamma distribution as the prior distribution for θ,Bayes estimators of $\phi(\theta)=$

$e^{-\frac{t}{\theta}}$ corresponding to various loss functions given as above and Bayes estimators of $w(\phi(\theta),\delta)$ based on loss functions as follows:

(a) When $w(\phi(\theta),\delta)=(\phi(\theta)-\delta)^2$

$$\widehat{\phi}_B(\theta)=(\frac{\beta+t_r}{\beta+t_r+t})^{(\alpha+r)} \quad (3.19)$$

$$\gamma_{\phi B}(\underline{X})=(\frac{\beta+t_r}{\beta+t_r+2t})^{(\alpha+r)}-(\frac{\beta+t_r}{\beta+t_r+t})^{2(\alpha+r)} \quad (3.20)$$

(b) When $w(\phi(\theta),\delta)=\delta^{-2}(\phi(\theta)-\delta)^2$

$$\widehat{\phi}_D(\theta)=(\frac{\beta+t_r+t}{\beta+t_r+2t})^{(\alpha+r)} \quad (3.21)$$

$$\gamma_{\phi D}(\underline{X})=1-\frac{(\beta+t_r)^{2(\alpha+r)}}{(\beta+t_r+t)^{2(\alpha+r)}} \quad (3.22)$$

(c) When $w(\phi(\theta),\delta)=\theta^{-2}(\phi(\theta)-\delta)^2$

$$\widehat{\phi}_M(\theta)=(\frac{\beta+t_r}{\beta+t_r+t})^{(\alpha+r+2)} \quad (3.23)$$

$$\gamma_{\phi M}(\underline{X})=\frac{(\beta+t_r)^{(\alpha+r)}\Gamma(\alpha+r+2)}{\Gamma(\alpha+r)}\{\frac{1}{(\beta+t_r+2t)^{(\alpha+r+2)}}-\frac{(\beta+t_r)^{(\alpha+r+2)}}{(\beta+t_r+t)^{2(\alpha+r+2)}}\} \quad (3.24)$$

(d) When $w(\phi(\theta),\delta)=\theta^{-2}e^{-a\theta^{-1}}(\phi(\theta)-\delta)^2$

$$\widehat{\phi}_E(\theta)=(\frac{\beta+t_r+a}{\beta+t_r+t+a})^{(\alpha+r+2)} \quad (3.25)$$

$$\gamma_{\phi E}(\underline{X})=\frac{(\beta+t_r)^{(\alpha+r)}\Gamma(\alpha+r+a+2)}{\Gamma(\alpha+r)}\{\frac{1}{(\beta+t_r+2t+a)^{(\alpha+r+2)}}-\frac{(\beta+t_r+a)^{(\alpha+r+2)}}{(\beta+t_r+t+a)^{2(\alpha+r+2)}}\} \quad (3.26)$$

Proof: The proof is similar to that in Theorem 1.

Remark: For r =n, we get results for the complete sample.

Definition: Let $\gamma_1(\underline{X})$ and $\gamma_2(\underline{X})$ be the two estimators based on Rukhin's loss function corresponding to two different forms of $w(\theta,\delta)$. $\gamma_1(\underline{X})$ is said to be dominate $\gamma_2(\underline{X})$ if $E_\theta\{\gamma_1(\underline{X})\}\le E_\theta\{\gamma_2(\underline{X})\}$

Since, $E_\theta\{\gamma_E(\underline{X})\}<E_\theta\{\gamma_M(\underline{X})\}<E_\theta\{\gamma_D(\underline{X})\}$, $\gamma_E(\underline{X})$ dominates $\gamma_M(\underline{X})$ while, $\gamma_M(\underline{X})$ dominates $\gamma_D(\underline{X})$ Since, $E_\theta\{\gamma_B(\underline{X})\}=\frac{r\theta^2+(r\theta+\beta)^2}{(r+\alpha-2)(r+\alpha-1)^2}>\frac{1}{(r+\alpha+1)}$, provided $\theta\ge 1$

Thus, $E_\theta\{\gamma_E(\underline{X})\}<E_\theta\{\gamma_M(\underline{X})\}<E_\theta\{\gamma_D(\underline{X})\}$,for all α , β ,and θ and $E_\theta\{\gamma_M(\underline{X})\}<E_\theta\{\gamma_B(\underline{X})$ for all β and $\theta\ge 1$.Hence, $\gamma_E(\underline{X})$ is most dominant among $\gamma_E(\underline{X}),\gamma_M(\underline{X}),\gamma_D(\underline{X})$ and $\gamma_B(\underline{X})$

4. Conclusion

In this paper Bayesian estimation of loss and risk functions for the unknown parameter θ of exponential distribution has been considered under Rukhin's loss function for three different forms of $w(\theta, \delta)$. The superiority of estimates has also been studied and it has been proved that when $w(\theta, \delta) = \theta^{-2} e^{-a\theta^{-1}} (\theta - \delta)^2$, $a > 0$,the corresponding estimate is most dominant.

5. Conflict of Interest

There exists no conflict of interest.

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